SOIL DYNAMICS AND MODELING

1. SOURCE OF DYNAMIC LOADING
2. WAVE PROPAGATION
3. DAMAGE DURING EARTHQUAKE
4. IDEALIZATION OF SOILS AND STRUCTURES FOR ANALYSIS
5. VIBRATION ANALYSIS
1. SOURCE OF DYNAMIC LOADING

- EARTHQUAKES
  Seismology, Epicenter-distance, Focal depth, Magnitude
  Modified Mercalli (MM) Scale
- PEAK GROUND MOTION
  H- and V- Ground motion
- FREQUENCY CONTENT
- RESPONSE OF HUMANS AND STRUCTURES TO VIBRATIONS
- DESIGN SEISMIC COEFFICIENTS

SOURCES OF CONSTRUCTION VIBRATIONS
(ORGANIZED BY TYPE)

- TYPES OF VIBRATIONS
  - Transient or Impact
    Blasting, Impact Pile Driving, Demolition.
  - Steady State (Continuous)
    Vibratory Pile Driver, Large Pumps, Compressors.
  - Pseudo Steady State
    Jack Hammers, Pavement Breakers, Trucks, etc.
• TYPICAL WAVE PROPAGATION CURVES

TYPES OF CONSTRUCTION VIBRATIONS

- Transient
- Steady-State
- Pseudo-Steady-State

V = Vibration Amplitude  T = Time

DIAGRAM SHOWING THE THREE MAIN TYPES OF FAULT MOTION

Strike-slip fault
Reverse Fault
Normal Fault
El Centro, Calif., earthquake of May 18, 1940, N-S component.

CORRECTED ACCELERATION, VELOCITY, DISPLACEMENT (M8.3) GUATEMALA EARTHQUAKES BETWEEN FEBRUARY 21 AND MAY 26, 1976 – SHOCK 1 CHICHICASTENANGO. SOUTH COMP PEAK VALUES ACCEL = -110.1 CM/SEC/SEC., VELOCITY = 5.635 CM/SEC., DISPL = 0.516 CM


Memphis Synthetic Ground Motion, Magnitude 6.3.

Richter Local Magnitude

In 1935, Charles Richter used a Wood – Anderson seismometer to define a magnitude scale for shallow, local (epicentral distances less than about 600 km (375 miles)) earthquakes in southern California (Richter, 1935).

Richter defined what is now known as the local magnitude as the logarithm (base 10) of the maximum trace amplitude (in micrometers) recorded on a Wood – Anderson seismometer located 100 km (62 miles) from the epicenter of the earthquake.

The Richter local magnitude (ML) is the best known magnitude scale, but it is not always the most appropriate scale for description of earthquake magnitude.
**Surface Wave Magnitude**

The Richter local magnitude does not distinguish between different types of waves. Other magnitudes scales that base the magnitude on the amplitude of a particular wave have been developed. At large epicentral distances, body waves have usually been attenuated and scattered sufficiently that the resulting motion is dominated by surface waves. The *surface wave magnitude* (Gutenberg and Richter, 1936) is a worldwide magnitude scale based on the amplitude of Rayleigh waves with a period of about 20 sec. The surface wave magnitude is obtained from:

\[ M_s = \log A + 1.66 \log \Delta + 2.0 \]

Where,
- \( A \) = maximum ground displacement in micrometers;
- \( \Delta \) = epicentral distance of the seismometer measured in degrees. 360 degrees corresponding to the circumference of the earth.

Note that the surface wave magnitude is based on the maximum ground displacement amplitude (rather than the maximum trace amplitude of a particular seismograph); therefore, it can be determined from any type of seismograph. The surface wave magnitude is most commonly used to describe the size of shallow (less than about 70 km (44 miles) focal depth), distant (farther than about 1000 km (622 miles)) moderate to large earthquakes.

*(Geotechnical Earthquake Engineering 1996, Steven L. Kramer)*

---

**Body Wave Magnitude**

For deep-focus earthquakes, surface waves are often too small to permit reliable evaluation of the surface wave magnitude. The *body wave magnitude* (Gutenberg, 1945) is a worldwide magnitude scale based on the amplitude of the first few cycles of p-waves which are not strongly influenced by the focal depth (Bolt, 1989). The body wave magnitude can be expressed as

\[ M_b = \log A - \log T + 0.01 \Delta + 5.9 \]

Where,
- \( A \) = p-wave amplitude in micrometers;
- \( \Delta \) = epicentral distance of the seismometer measured in degrees. 360 degrees corresponding to the circumference of the earth;
- \( T \) = period of the p-wave (usually about one sec). Body wave magnitude can also be estimated from the amplitude of one-second-period, higher-mode Rayleigh waves (Nuttli, 1973); the resulting magnitude, \( M_{blg} \), is commonly used to describe intraplate earthquakes.

*(Geotechnical Earthquake Engineering 1996, Steven L. Kramer)*
MODIFIED MERCALLI INTENSITY SCALE OF 1931

I  Not felt except by a very few under especially favorable circumstances
II  Felt by only a few persons at rest, especially on upper floors of buildings; delicately suspended objects may swing
III  Felt quite noticeably indoors, especially on upper floors of buildings, but many people do not recognize it as an earthquake; standing motor cars may rock slightly; vibration like passing of truck; duration estimated
IV  During the day felt indoors by many, outdoors by few; at night some awakened; dishes, windows, doors disturbed; walls make cracking sound; sensation like heavy truck striking building; standing motor cars rocked noticeably
V  Felt by nearly everyone, many awakened; some dishes, windows, etc., broken; a few instances of cracked plaster; unstable objects overturned; disturbances of trees, piles, and other tall objects sometimes noticed; pendulum clocks may stop
VI  Felt by all, many frightened and run outdoors; some heavy furniture moved; a few instances of fallen plaster or damaged chimneys; damage slight

Cont., MODIFIED MERCALLI INTENSITY SCALE OF 1931

VII  Everybody runs outdoors; damage negligible in buildings of good design and construction, slight to moderate in well-built ordinary structures, considerable in poorly built or badly designed structures; some chimneys broken; noticed by persons driving motor cars
VIII  Damage slight in specially designed structures, considerable in ordinary substantial buildings, with partial collapse, great in poorly built structures; panel walls thrown out of frame structures; fall of chimneys, factory stacks, columns, monuments, walls; heavy furniture overturned; sand and mud ejected in small amounts; changes in well water; persons driving motor cars disturbed
IX  Damage considerable in specially designed structures; well-designed frame structures thrown out of plumb; great in substantial buildings, with partial collapse; buildings shifted off foundations; ground cracked conspicuously; underground pipes broken
X  Some well-built wooden structures destroyed; most masonry and frame structures destroyed with foundations; ground badly cracked; rails bent; landslides considerable from river banks and steep slopes; shifted sand and mud; water splashed over banks
XI  Few, if any (masonry) structures remain standing; bridges destroyed; broad fissures in ground; underground pipelines completely out of service; earth slumps and land slips in soft ground; rails bent greatly
XII  Damage total; practically all works of construction are damaged greatly or destroyed; waves seen on ground surface; lines of sight and level are distorted; objects thrown into the air
Northridge earthquake of Jan 17, 1994, 90 degree component (M6.7): a) accelerogram, b) Fourier’s spectrum with predominant frequency.

El-Centro earthquake of May 18, 1940, SE component (M7.1): a) accelerogram, b) Fourier’s spectrum with predominant frequency.
Loma–Prieta Earthquake (M 7.0), Oct. 17, 1989, Diamond Heights (M7.0): a) accelerogram, b) Fourier’s spectrum with predominant frequency.

Seismograph or accelerogram record produced by seismograph
Relative energy of various natural and human-made phenomena.

(After Johnston, 1990.)

Limiting amplitudes of vibrations for a particular frequency.

(After Richart, 1962)

- From Reihm & Melstar (1933) - steady state vibrations
- From Rausch (1943) - steady state vibrations
- From Overbend (1945) - due to blasting
Criteria for vibrations of rotating machinery.

Explanations of classes:

**AA Dangerous.** Shut it down now to avoid danger.

**A Failure is near.** Correct within two days to avoid breakdown.

**B Faulty.** Correct it within 10 days to save maintenance dollars.

**C Minor Faults.** Correction wastes dollars

**D No faults.** Typical new equipment.

This is a guide to aid judgment, not to replace it. Use common sense. Use with care. Take account of all local circumstances. Consider: safety, labor costs, downtime costs. (After Blake, 1964.)

---

**NEHRP Coefficients Aa and Av**

<table>
<thead>
<tr>
<th>Map Area from Map 1 (for $A_u$) or Map 2 (for $A_v$)</th>
<th>Value of $A_u$ and $A_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.40</td>
</tr>
<tr>
<td>6</td>
<td>0.30</td>
</tr>
<tr>
<td>5</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
</tr>
<tr>
<td>1</td>
<td>&lt; 0.05&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<sup>a</sup> For equations or expressions incorporating the terms $A_u$ or $A_v$, a value of 0.05 shall be used.

For equations or expressions incorporating the terms $A_a$ or $A_v$ a value of 0.05 shall be used.
Maps of NEHRP seismic loading zones:

(a) map 1 for Aa

Maps of NEHRP seismic loading zones:

(b) map 2 for Av
SOIL DYNAMICS AND MODELING

1. SOURCE OF DYNAMIC LOADING
2. WAVE PROPAGATION
3. DAMAGE DURING EARTHQUAKE
4. IDEALIZATION OF SOILS AND STRUCTURES FOR ANALYSIS
5. VIBRATION ANALYSIS

2. WAVE PROPAGATION

- P – WAVES
- S – WAVES
- R – WAVES
- BODY AND SURFACE WAVES

WAVE ISOLATION
- ACTIVE, PASSIVE
1. COMPRESSION (P) - WAVES

Direction of wave travel and particle motions are the same

\[ V_p = \sqrt{\frac{E(1 - \nu)}{\rho(1 + \nu)(1 - 2\nu)}} \]

2. SHEAR (S) - WAVES

Direction of wave travel and particle motions are perpendicular to each other

\[ V_s = \sqrt{\frac{G}{\rho}} \]

3. RAYLEIGH (R) - WAVE

Direction of particle motion is in 2-perpendicular directions to the direction of wave travel

\[ V_p > V_s \]
\[ V_c < V_r \]
Wave system at a point from surface point source in ideal medium.

Variation of Rayleigh wave and body wave propagation velocities with Poisson’s ratio.
RADIATION DAMPING WAVE PROPAGATION IN AN ELASTIC MEDIUM

Distribution of displacement waves from a circular footing on a homogenous, isotropic, elastic half space (Woods, 1968)

<table>
<thead>
<tr>
<th>Wave Type</th>
<th>Percent of total energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh</td>
<td>67</td>
</tr>
<tr>
<td>Shear</td>
<td>26</td>
</tr>
<tr>
<td>Compression</td>
<td>7</td>
</tr>
</tbody>
</table>

Variation of shear wave velocity and shear modulus with void ratio and confining pressure for dry round and angular-grained sands.
Horizontal and vertical motion of Rayleigh waves. A negative amplitude ratio indicates that the displacement is in the opposite direction of the surface displacement.

P and S-wave Velocities

<table>
<thead>
<tr>
<th>Category</th>
<th>$S$, MPa $^2$</th>
<th>$\nu$</th>
<th>$V_p$, m/s</th>
<th>$V_s$, m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mudclay</td>
<td>400-410</td>
<td>0.30</td>
<td>1100</td>
<td>150</td>
</tr>
<tr>
<td>Loess at natural moisture</td>
<td>—</td>
<td>0.44</td>
<td>810</td>
<td>260</td>
</tr>
<tr>
<td>Loose sand and gravel</td>
<td>45</td>
<td>0.30-0.35</td>
<td>310</td>
<td>110</td>
</tr>
<tr>
<td>Fine-grained sand</td>
<td>70</td>
<td>0.30-0.35</td>
<td>310</td>
<td>110</td>
</tr>
<tr>
<td>Medium-grained sand</td>
<td>100</td>
<td>0.30-0.35</td>
<td>310</td>
<td>110</td>
</tr>
<tr>
<td>Medium-stone gravel</td>
<td>200</td>
<td>0.30-0.35</td>
<td>310</td>
<td>110</td>
</tr>
<tr>
<td>Rubber</td>
<td>1.87</td>
<td>0.50</td>
<td>43</td>
<td>77</td>
</tr>
<tr>
<td>Glass</td>
<td>55000</td>
<td>0.25</td>
<td>5300</td>
<td>3330</td>
</tr>
<tr>
<td>Copper</td>
<td>100000</td>
<td>0.24</td>
<td>3670</td>
<td>2250</td>
</tr>
<tr>
<td>Aluminum</td>
<td>69000</td>
<td>0.34</td>
<td>5000</td>
<td>3100</td>
</tr>
<tr>
<td>Steel</td>
<td>210000</td>
<td>0.29</td>
<td>5000</td>
<td>3220</td>
</tr>
</tbody>
</table>

P-wave Velocities in Rocks

<table>
<thead>
<tr>
<th>Type of rock</th>
<th>Velocity $V_p$, km/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand, gravel, silt</td>
<td>0.5–2.0</td>
</tr>
<tr>
<td>Shale, sandstone</td>
<td>1.5–4.5</td>
</tr>
<tr>
<td>Limestone</td>
<td>3.0–5.2</td>
</tr>
<tr>
<td>Dolomite</td>
<td>4.3–6.0</td>
</tr>
<tr>
<td>Granite</td>
<td>4.0–5.5</td>
</tr>
<tr>
<td>Basalt, gabbro</td>
<td>4.3–6.0</td>
</tr>
</tbody>
</table>
Schematic of vibration isolation using a circular trench surrounding the source of vibrations—active isolation.

Schematic of vibration isolation using a straight trench—passive isolation.
SOIL DYNAMICS AND MODELING

1. SOURCE OF DYNAMIC LOADING
2. WAVE PROPAGATION
3. DAMAGE DURING EARTHQUAKE
4. IDEALIZATION OF SOILS AND STRUCTURES FOR ANALYSIS
5. VIBRATION ANALYSIS

3. DAMAGE DURING EARTHQUAKE

- DAMAGE DUE TO LIQUEFACTION
- DAMAGE TO PILES
- MEXICO EARTHQUAKE
- LIFE LINES
- SURFACE FAULTING
- DAMAGE TO DAMS
DAMAGE DUE TO LIQUEFACTION

SAND BOILS
SAND BOILS

Tilting of Buildings in Niigata (Japan) 1964
Tilting of about 15 degrees

Another Tilted Building
A Septic Tank Moves Above Ground in Niigata

Failure of 4 Spans of Niigata Bridge 1964
Tilt of Building in Guatemala EQ 1976

Liquefaction During 2010 Haiti Earthquake
Liquefaction During 2010 Haiti Earthquake

Mexicali 7.2 Earthquake on Rio Hardy, Mexico: The small river community is located approximately 40 miles southeast of Mexicali and estimated less than 5 miles from the epicenter. (2010)
Mexicali 7.2 Earthquake on Rio Hardy, Mexico: The small river community is located approximately 40 miles southeast of Mexicali and estimated less than 5 miles from the epicenter. (2010)

Fig. 7  Map of Eastern San Francisco Showing the Region Most Intensively Damaged During the 1906 Earthquake (Before the Post-Earthquake Fire), and the Historic Coastline and Marshes of 1852
CONCLUSION:

Liquefaction can occur at the same site again as in San Francisco 1906 and 1989.
Liquefaction in Loma Prieta
Earthquake 1989

Fig. 1 Map of Affected Region
Showing Sites of Soil Liquefaction

Tsunami (maximum reported run up height of 38 m)

Photo by Reuters (framework.latimes.com)
107,000 partial collapse/collapse & 230,000 damaged homes (Police – May 1, 2011).
15,421 dead, 5,367 injured, & 7,937 missing (Police – June 5, 2011)

92% of victims drowned; 65% were >60 yrs old (Yomiuri Shimbun 4/19/11; Courtesy L. Johnson)
Response hindered by damages to roads, railways, airports, and port; e.g., first relief flights from Sendai airport were on March 17th.

Loss of 561 km² (138,000 acres) along coast (Geospatial Info. Authority of Japan; L. Johnson)
25 mil tons of debris will take 3 years to clean up (Japan Times 4/2011)

Fukushima Dai-ichi nuclear power plant

Photo by Adrees Latif/Reuters

Photo: DigitalGlobal, via Agence France-Presse – Getty Images
Liquefaction in loose reclaimed land (a known pervasive hazard)

Damage to Dams
Naruse river - Levee & approach road at km 30
Fujinuma Auxiliary Dam – Upstream Slide

DAMAGE TO PILES
Damage to pile by 2m of lateral ground displacement during 1964 Niigata earthquake (Yosuda et al.1999)

Detailed profiles of the quay wall movement and ground distortion in the backfills at Section M-5 (Ishihara and Cubrinovski, 2004)
Lateral displacement and observed cracks on the inside wall of Pile No. 9 Kobe 1995 EQ (Ishihara and Cubrinovski, 2004)

Lateral displacement and observed cracks on the inside wall of Pile No. 2 Kobe 1995 EQ (Ishihara and Cubrinovski, 2004)
Lateral ground displacement versus distance from the waterfront along Section M-5, Kobe 1995 EQ (Ishihara and Cubrinovski, 2004)

MEXICO EARTHQUAKE
Figure 2- Schematic section showing relative locations of the epicentral station at Caleta de Campos, Teacalco station (closest to Mexico City), and Mexico City Stations, UNAM (hills zone) and SCT (lake zone). The seismograms are east-west components of 19 September 1985 acceleration time-histories (all plotted to the same scale) recorded at retrospective stations and demonstrate the attenuation of motions with distance from the coast as well as amplification of motions at the lakebed of Mexico City.

Northridge earthquake of Jan 17, 1994, 90 degree component (M6.7):
a) accelerogram, b) Fourier’s spectrum with predominant frequency.
El-Centro earthquake of May 18, 1940, SE component (M7.1):
a) accelerogram, b) Fourier’s spectrum with predominant frequency.

Loma-Prieta Earthquake (M 7.0), Oct. 17, 1989, Diamond Heights (M7.0):
a) accelerogram, b) Fourier’s spectrum with predominant frequency.
CONCLUSION

Double amplification had been observed in a significant manner first time in an earthquake:

a) From rock to soft soil surface
b) From soil surface to top of the building
c) Dominant frequency of ground motion controls damage to buildings in a significant way
PREFAB Construction
(Mexico 1985)

SURFACE FAULTING
Orange County (CA)
Break in (about 8 feet) Wall about 8'

Soft Story Effect

Figure 3-2: Failed columns in the Imperial County Service Building, 1979. (Photo: Helmut Krawäcker)
SOIL DYNAMICS AND MODELING

1. SOURCE OF DYNAMIC LOADING
2. WAVE PROPAGATION
3. DAMAGE DURING EARTHQUAKE
4. IDEALIZATION OF SOILS AND STRUCTURES FOR ANALYSIS
5. VIBRATION ANALYSIS

4. IDEALIZATION OF SOIL STRUCTURES FOR ANALYSIS

• DISCRETE SYSTEMS
• DISTRIBUTED MASSES SYSTEMS
DISCRETE SYSTEM

Mathematical model of rigid block embedded in elastic half-space with soil side layer in coupled motion

Equivalent formulations of inertial interaction analysis for structures with rigid foundation

Inertia forces applied to each element

Foundation motion applied through frequency-dependent springs and dashpots (not shown)
Typical n–story frame

Mathematical model for a single story frame on flexible foundation

Deflected shape of a single story frame
Mathematical model for two story frame with flexible foundation

Deflected shape of a story frame
Direct method of soil–structure interaction analysis. Entire problem is modeled and response to free–field motion applied at boundaries is determined in a single step.

TYPICAL HIGHWAY BRIDGE ABUTMENT SUPPORTED ON PILES
TRANSLATION AND ROTATION MOVEMENT OF ABUTMENT

a. Initial Condition  

b. Sliding  

c. Sliding and Rotation

FORCES ACTING ON THE BRIDGE ABUTMENT

a) Static forces  

b) Dynamic force increments
Plan and Cross Section of Pile Group

SIGN CONVENTIONS
EIGHT SPRING CONSTANTS

\[ k_x, k_y, k_z \quad \text{TRANSLATION} \]
\[ k_\theta, k_\phi, k_\psi \quad \text{ROTATION} \]
\[ k_x\phi, k_y\theta \quad \text{CROSS-COUPLING} \]

EIGHT DAMPING CONSTANTS

\[ c_x, c_y, c_z \quad \text{TRANSLATION} \]
\[ c_\theta, c_\phi, c_\psi \quad \text{ROTATION} \]
\[ c_x\theta, c_y\phi \quad \text{CROSS-COUPLING} \]

SOIL DYNAMICS AND MODELING

1. SOURCE OF DYNAMIC LOADING
2. WAVE PROPAGATION
3. DAMAGE DURING EARTHQUAKE
4. IDEALIZATION OF SOILS AND STRUCTURES FOR ANALYSIS
5. VIBRATION ANALYSIS
5. VIBRATION ANALYSIS

- SPRING – MASS – DASHPOT SYSTEM
- NATURAL FREQUENCY
- DAMPING: NATURE OF DAMPING
  - Viscous Damping
  - Friction Damping
  - Radiation Damping
  - Total Damping
- SINGLE DEGREES OF FREEDOM SYSTEM (SDOF)
- TWO DEGREES OF FREEDOM SYSTEM (2DOF)
- MULTI DEGREES OF FREEDOM SYSTEM (MDOF)
- CONCLUDING REMARKS

IMPORTANT DEFINITIONS

- NATURAL FREQUENCY
- DEGREES OF FREEDOM
- DAMPING
- CRITICAL DAMPING
- ORDER OF DAMPING IN MATERIALS
- ORDER OF DAMPING IN STRUCTURES
  - LINEAR DAMPING
  - NON LINEAR DAMPING
A. THEORY OF VIBRATIONS

Simple theoretical concepts of harmonic vibrations

B. DEFINITIONS

PERIOD: If motion repeats itself in equal intervals of time, it is called a periodic motion and the time elapsed in repeating the motion once is called its period.

CYCLE: Motion completed during a period is referred to as a cycle.

FREQUENCY: The number of cycles of motion in a unit of time is called the frequency of vibration.

NATURAL FREQUENCY: If an elastic system vibrates under the action of forces in the system and in the absence of any externally applied force, the frequency with which it vibrates is its natural frequency.

FORCED VIBRATIONS: Vibrations that occur under the excitation of external forces are termed forced vibrations. Forced vibrations occur at a frequency of the exciting force. The frequency of excitation is independent of the natural frequency of the system.

DEGREES OF FREEDOM: The number of independent coordinates necessary to describe the motion of a system specifies the degrees of freedom of the system. A system may in general have several degrees of freedom; such a system is called a multidegree freedom system.

One degree of freedom (n=1)  Two degrees of freedom (n=2)
Three degrees of freedom \( (n=3) \)  Infinite degrees of freedom \( (n=\infty) \)

DAMPING

Damping force \( (F_d) \) is resistance to motion of an oscillating system.

1. Viscous damping \( (C) \)
   \[ F_d = C \cdot \dot{x} \]
   where, \( C = \text{coefficient of viscous damping} \)

2. Friction damping \( (C_f) \)
   \[ F_d = W \cdot \mu \]
   where, \( \mu = \text{coefficient of friction} \)

3. Material damping \( (C_m) \)
   \[ F_d = C_m \cdot \ddot{x} \]

4. Radiation or geometrical damping \( (C_r) \)
   Energy dissipated in an ELASTIC half space.

5. Total damping
   \[ C_{total} = C_{viscous} + C_{friction} + C_{material} \]

6. Critical damping \( C_c \)
   A physical condition where a system does not oscillate if disturbed from equilibrium and returns to original position in minimum time. \( t = \frac{1}{2} t_{max} \)

   \[ C > C_c, \text{The system still non-oscillatory} \]

7. Damping factor
   \[ \xi = \frac{C_{friction}}{C_c} \]
RADIATION DAMPING WAVE PROPAGATION IN AN ELASTIC MEDIUM

Distribution of displacement waves from a circular footing on a homogenous, isotropic, elastic half space (Woods, 1968)

Mass ratio B, damping factor $\zeta$, and spring constant $k$ for rigid circular footing on the semi– (static $k'$) infinite elastic half space

<table>
<thead>
<tr>
<th>Mode of vibration</th>
<th>Mass (or inertia) ratio</th>
<th>Damping factor $\zeta$</th>
<th>Spring constant $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>$B_v = \frac{(1 - \nu) m}{4 \rho R^2}$</td>
<td>$\frac{0.425}{\sqrt{B_v}}$</td>
<td>$k_v = \frac{4G\rho}{1 - \nu}$</td>
</tr>
<tr>
<td>Sliding</td>
<td>$B_s = \frac{(7 - 8\nu) m}{32(1 - \nu) \rho R^2}$</td>
<td>$\frac{0.2875}{\sqrt{B_s}}$</td>
<td>$k_s = \frac{32(1 - \nu) G\rho}{7 - 8\nu}$</td>
</tr>
<tr>
<td>Rocking</td>
<td>$B_p = \frac{3(1 - \nu)}{8} \frac{M_{ho}}{\rho R^2}$</td>
<td>$\frac{0.15}{1 + B_p \sqrt{B_p}}$</td>
<td>$k_p = \frac{8G\rho^2}{3(1 - \nu)}$</td>
</tr>
<tr>
<td>Torsional</td>
<td>$B_t = \frac{J_t}{\rho R^2}$</td>
<td>$\frac{0.5}{1 + 2B_t}$</td>
<td>$k_t = \frac{16G\rho^2}{3}$</td>
</tr>
</tbody>
</table>
RADIATION DAMPING IN SOILS

Equivalent damping ratio for oscillation of rigid circular footing on the elastic half–space

DEGREES OF FREEDOM OF A BLOCK FOUNDATION

SIX DEGREES OF FREEDOM:
1. Translation along Z axis
2. Translation along X axis
3. Translation along Y axis
4. Rotation along Z axis
5. Rotation along X axis
6. Rotation along Y axis

Coupled Motion:
- a. 2 and 6
- b. 3 and 5

Modes of vibration of a rigid block foundation
TYPES OF UNBALANCED LOADS OF MACHINES ON FOUNDATIONS

- Pure vertical translation
- Pure rocking
- Simultaneous horizontal sliding and rocking
- Pure torsional oscillations

DEFINITION OF SOIL SPRING STIFFNESS

a. Uniform compression
b. Uniform shear
c. Non-uniform compression
d. Non-uniform shear

Therefore, the soil constant characterizing the stress below the block and the corresponding elastic deformation are different in each case:

- Vertical Vibrations
- Horizontal Translations
- Rocking
- Yawing
METHODS OF ANALYSIS

**Figure 5. Vertical Vibrations of a Machine Foundation**

(a) Actual case, (b) Equivalent model with damping, (c) Model without damping

---

**Elastic-half –space -analogs**

**Surface Foundations**

*Vertical vibrations:* The problem of vertical vibrations is idealized as a single degree freedom system with damping as shown in Fig. 13.15b. Hsieh (1962) and Lysmer and Richart (1966) have provided a solution. The equation of vibration is:

\[ m \ddot{z} + \frac{3.4r_s^2}{(1-v)^2} \nu \rho G \dot{z} + \frac{4Gr_s^2}{(1-v)^2} z = P \sin(\omega t) \]

Where \( r_s \) = radius of the foundation. (For non-circular foundations, appropriate equivalent radius may be used, see Eqs. 40-42).
The equivalent spring for vertical vibrations is given by

\[ k_x = \frac{4Gr_0}{1 - \nu} \]

And the damping \( c_x \) is given by

\[ c_x = \frac{3.4r_0}{1 - \nu} \sqrt{\rho G} \]

The damping constant for vertical vibrations \( \xi_x \) is given by

\[ \xi_x = \frac{0.425}{\sqrt{B_x}} \]

In which \( B_x \) is known as the modified mass ratio, given by

\[ B_x = \frac{1 - \nu}{4} \frac{m}{\rho r_0^2} \]

The undamped natural frequency of vertical vibrations may now be obtained using Eqs. 6 and 7.

\[ \omega_{ns} = \frac{k_x}{\sqrt{m}} \]

\[ f_{ns} = \frac{1}{2\pi} \frac{k_x}{m} \]

In which \( \omega_{ns} \) is the circular natural frequency (undamped) of the soil foundation system in vertical vibration (rad/sec) and \( f_{ns} \) = natural frequency of vertical vibrations (Hz).

The amplitude of vertical vibration is obtained as:

\[ A_x = \frac{P_t}{k_x \sqrt{(1 - r^2)^2 + (2\xi_x r)^2}} = \frac{P_t}{k_x \left[ \left(1 - \frac{\omega}{\omega_{ns}}\right)^2 + \left(\frac{2\xi_x \omega}{\omega_{ns}}\right)^2 \right]^{1/2}} \]
Sliding vibrations

The equation of the analog for sliding is (Fig. 6)

\[ m \ddot{x} + c_x + k_x x = P_x \sin(\omega t) \]

Figure 6. Sliding Vibrations of a Rigid Block (a) Actual case (b) Equivalent model

Hall (1967) defined the modified mass ratio for sliding as:

\[ B_x = 7 - 8\nu \frac{m}{32(1-\nu) \rho r_o^3} \]

where \( r_o \) = radius of the foundation.

The expressions for the equivalent spring and damping factors are as follows:

The equivalent spring

\[ k_x = \frac{32(1-\nu)}{7 - 8\nu} G r_o \]

And the equivalent damping

\[ c_x = \frac{18.4(1-\nu)}{7 - 8\nu} r_o^2 \rho \bar{G} \]

The damping ratio \( \xi_x \) is given by

\[ \xi_x = \frac{c_x}{c_e} = \frac{0.2875}{\sqrt{B_x}} \]

The undamped natural frequency of sliding vibration may be obtained as follows:

\[ \omega_{nx} = \frac{k_x}{m} \]

\[ f_{nx} = \frac{1}{2\pi} \sqrt{\frac{k_x}{m}} \]
Rocking Vibrations: A rigid block foundation undergoing rocking vibrations due to an exciting moment $M$, $\sin\omega t$ is shown in Fig. 7.

Hall (1967) proposed an equivalent mass-spring-dashpot model that can be used to determine the natural frequency and amplitude of vibration of a rigid circular footing resting on an elastic half-space and undergoing rocking vibrations (Fig. 7). The equivalent model is given in equation 16

$$M_{m0}\ddot{\phi} + c_\phi \dot{\phi} + k_\phi \phi = M_t \sin(\omega t)$$

In which $k_\phi$ = spring constant for rocking, $c_\phi$ = damping constant and $M_{m0}$ = mass moment of inertia of the foundation and machine about the axis of rotation through the base.

$$M_{m0} = M_m + mL^2$$

Where $M_m$ = mass moment of inertia of foundation and machine about an axis passing through the centroid of the system and parallel to the axis of rotation and $L$ = the height of the centroid above the base.

The terms $k_\phi$ and $c_\phi$ can be obtained as follows:

$$k_\phi = \frac{8Gt^3}{3(1 - \nu)}$$

$$c_\phi = \frac{0.8r^4}{(1 - \nu)(1 + B_\phi)}$$

In which $r_\phi = \text{radius}$. 

In which $\omega_n$ = the circular natural frequency (undamped) in sliding vibrations and $f_n$ = natural frequency of sliding vibrations (Hz).

The damped amplitude in sliding is obtained as:

$$A_x = \frac{P}{k_x \sqrt{(1 - \left(\frac{\omega}{\omega_{nx}}\right)^2) + (2\xi_n \frac{\omega}{\omega_{nx}})^2}}$$
$B_\phi$, in Eq. 19, is known as the modified inertia ratio which obtained as follows:

$$B_\phi = \frac{3(1 - v) M_{ma}}{8 \rho r^2}$$

**Figure 7. Rocking vibrations of a rigid block under excitation due to an applied moment**

The damping factor $\xi_\phi$ is given by

$$\xi_\phi = \frac{c_\phi}{k_\phi} = \frac{0.15}{(1 + B_\phi) \sqrt{B_\phi}}$$

The undamped natural frequency of rocking

$$\omega_{n,\phi} = \frac{k_\phi}{M_{ma}} \text{ rad/sec}$$

Damped amplitude of rocking vibrations $A_\phi$ is given by Eq. 23

$$A_\phi = \frac{M_\phi}{k_\phi \sqrt{1 - \left(\frac{\omega}{\omega_{n,\phi}}\right)^2 + \left(2\xi_\phi \frac{\omega}{\omega_{n,\phi}}\right)^2}}$$
Torsional vibrations: A block foundation undergoing torsional vibrations is shown in Fig. 8. Non-uniform shearing resistance is mobilized during such vibrations. The analog solution for torsional vibrations is provided by Richard et al., (1970).

The equation of motion is

\[ M_{\text{rot}} \Psi + C_\psi \Psi + k_\psi \Psi = M_{\text{ext}} e^{\Psi t} \]

In which \( M_{\text{rot}} = \) mass moment of inertia of the machine and foundation about the vertical axis of rotation (polar mass moment of inertia). The spring constant \( k_\psi \) and the damping constant \( c_\psi \) are given by (Richart and Whitman, 1967):

\[ k_\psi = \frac{16}{3} Gr^3 \]

\[ c_\psi = \frac{1.6 \gamma_{\text{aeq}} Gp}{1 + B_\psi} \]

where \( r_{\text{eq}}(r_{\psi}) = \) equivalent radius.
Combined rocking and sliding: The problem of combined rocking and sliding is shown schematically in Fig. 9. The equations of motion are written as:

\[ m\ddot{x} + c_x\dot{x} + k_x x - Lc_x\phi - Lk_x\dot{\phi} = P_x e^{int} \]  \hspace{1cm} (31)

\[ M_m\ddot{\phi} + (c_\phi + L^2 C_x)\dot{\phi} + (k_\phi + L^2 k_x)\phi - Lc_x\dot{x} - Lk_x x = M_x e^{int} \]  \hspace{1cm} (32)

The undamped natural frequencies for this case can be obtained from Eq. 33.

\[ \omega_n^2 - \frac{\omega_{nx}^2 - \omega_{np}^2}{\gamma} \omega_n^2 + \frac{\omega_{nx}^2 \cdot \omega_{np}^2}{\gamma} = 0 \]  \hspace{1cm} (33)

In which

\[ \gamma = \frac{M_m}{M_{mx}} \]  \hspace{1cm} (34)
The damping in rocking and sliding modes will be different. Prakash and Puri (1988) developed equations for determination of vibration amplitudes for this case. Damped amplitudes of rocking and sliding occasioned by an exciting moment $M_r$ can be obtained as follows:

$$A_x = \frac{M_x}{M_m} \frac{\left[(\omega_{ax}^2)^2 + (2\xi_x \omega \omega_{m}^2)^2\right]^{1/2}}{\Delta(\omega^2)}$$

$$A_y = \frac{M_y}{M_m} \frac{\left[(\omega_{ay}^2 - \omega^2)^2 + (2\xi_y \omega \omega_{m}^2)^2\right]^{1/2}}{\Delta(\omega^2)}$$

The value of $\Delta(\omega^2)$ is obtained from Eq. 38

$$\Delta(\omega^2) = \left[ \omega^4 - \omega_{m}^4 + \frac{\omega_{ax}^4 + \omega_{ay}^4}{\gamma} - \frac{4\xi_x \omega \omega_{m}^2}{\gamma} \right]^{1/2} + \left[ \frac{\xi_x \omega_{ax}^2}{\gamma} (\omega_{ax}^4 - \omega^2) + \frac{\xi_y \omega_{ay}^2}{\gamma} (\omega_{ay}^4 - \omega^2) \right]^{1/2}$$
Damped amplitudes of rocking and sliding occasioned by a horizontal force \( F \) are given by Eqs. 38 and 39

\[
A_z = \frac{F_z}{mM_m} \left[ (1 - M_m \omega^2 + k_p + L^2 k_z)^2 + 4 \omega^2 (\xi_p M_m + \xi_z m)^2 \right]^{1/2}
\]

And

\[
A_\phi = \frac{F_L \omega_m (\omega_{\phi m}^2 + 4 \xi_{\phi m}^2)^{1/2}}{\Delta \omega^2}
\]

In case the footing is subjected to the action of a moment and a horizontal force, the resulting amplitudes of sliding and rocking may be obtained by adding the corresponding solutions from Eqs. 35, 36, 38 and 39.

---

**Table 3. Value of equivalent spring and damping constants for embedded foundations (Beredugo and Novak 1972, Novak and Beredugo 1972, Novak and Sachs 1973)**

<table>
<thead>
<tr>
<th>Mode of Vibration</th>
<th>Equivalent spring</th>
<th>Equivalent Damping constant</th>
<th>Damping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>( k_u = G \left[ \frac{G}{r} + \frac{b}{r} \right] )</td>
<td>( c_u = \sqrt{G \rho} \left[ \frac{G}{r} + \frac{b}{r} \right] )</td>
<td>( \xi_{xx} = \frac{c_x}{2M \omega_{xx}} )</td>
</tr>
<tr>
<td>Sliding</td>
<td>( k_u = G \left[ \frac{G}{r} + \frac{b}{r} \right] )</td>
<td>( c_u = \sqrt{G \rho} \left[ \frac{G}{r} + \frac{b}{r} \right] )</td>
<td>( \xi_{xx} = \frac{c_x}{2M \omega_{xx}} )</td>
</tr>
<tr>
<td>Rocking</td>
<td>( k_u = G \left[ \frac{G}{r} + \frac{b}{r} \right] )</td>
<td>( c_u = \sqrt{G \rho} \left[ \frac{G}{r} + \frac{b}{r} \right] )</td>
<td>( \xi_{xx} = \frac{c_x}{2M \omega_{xx}} )</td>
</tr>
<tr>
<td>Torsional or Yawing</td>
<td>( k_u = G \left[ \frac{G}{r} + \frac{b}{r} \right] )</td>
<td>( c_u = \sqrt{G \rho} \left[ \frac{G}{r} + \frac{b}{r} \right] )</td>
<td>( \xi_{xx} = \frac{c_x}{2M \omega_{xx}} )</td>
</tr>
</tbody>
</table>

The values of frequency independent parameters \( r, \rho \) for the elastic zone are given in Table 4. The values of frequency dependent parameters \( \xi_{xx} \) is for the elastic zone are given in Table 4.

\( r, \rho \) refer to radius and depth of embedment of the foundation respectively.

---

**Table 3.** Value of equivalent spring and damping constants for embedded foundations (Beredugo and Novak 1972, Novak and Beredugo 1972, Novak and Sachs 1973)
Table 4. Computation response of an embedded foundation by elastic half-space method for coupled rocking and sliding (Beredugo and Novak 1972).

| Mode of | Frequency-constant term $C_x$ | Frequency-constant term $C_x$ | Frequency-constant term $C_x$ | Damping coefficient $D_x$ | Various rocking equations
|---------|-----------------------------|-----------------------------|-----------------------------|--------------------------|--------------------------
| Sliding | 0.5                         | 0.5                         | 0.5                         | 0.10                     | 10.92                    |
| Yawing  | 0                           | 0                           | 0                           | 0.2          | 3.41                     |
|         |                             |                             |                             |                          |                          |

\[
\text{Equation} \quad \frac{\partial^2 \psi}{\partial t^2} + 2\zeta \omega_0 \frac{\partial \psi}{\partial t} + \omega_0^2 \psi = 0
\]
CONCLUSION

• Dynamic loads are frequency dependent.
• During Earthquake, frequency of ground motion is time dependent.
• Liquefaction is major type of damage in saturated soft soils.
• Liquefaction may occur at same site in more than one event.
• Double amplification was observed characteristically during 1985 Mexico earthquake.
• More Importantly –New events may teach us new knowledge which may be used for safer design of safer structures.

THANK YOU
QUESTIONS?

REFERENCES:


A.A. Balkema, Rotterdam
Bibliography

- Boulanger_EERI_Lessons_April_10_2012
- Boulanger_CalGeo_Tohoku_earthquake_June_19_2012
- Guinness Book of World Records

- Richart, (1966)